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# RESPLENDENCY AND RECURSIVE DEFINABILITY IN $\omega$-STABLE THEORIES

BY

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*Dedicated to the memory of Abraham Robinson on the tenth anniversary of his death*

ABSTRACT

We prove Theorem A. *Every resplendent model of an $\omega$-stable theory is homogeneous.* As an application we obtain Theorem B. *Suppose $T$ is $\omega$-stable, $M \models T$ is recursively saturated and $p \in S(M)$ is such that for all finite $\bar{m} \in M$, $p \restriction \bar{m}$ is realized in $M$. Then there is a $\bar{c} \in M$ and a definition $d$ of $p$ over $\bar{c}$ such that $d$ is recursive in $t(\bar{c}/\varnothing)$.*

In [1] we proved a "non-recursive" version of Theorem A: *Every relation-universal model of an $\omega$-stable theory is saturated.* We also showed that Theorem A follows from Theorem B. In this paper we give a direct proof of Theorem A. Theorem B then follows from results found in [2] and [3]. We refer the reader to [1] for the motivation behind the theorems of this paper.

Throughout this paper $T$ will denote an $\omega$-stable theory in a finite language. For the most part, our notation is as found in [5]. As in [1], if $c$ is a finite sequence in the monster model, we let $T(c)$ denote the theory in a language containing constants for $c$ which consists of $\{\varphi(c): \models \varphi(c), \varphi \in L\}$. Finite sequences of elements are denoted by $a, b, \ldots$, or by $\bar{a}, \bar{b}, \ldots$, when we want to emphasize that they are sequences. Both $\varphi(b)$ and $\models \varphi(b)$ mean that $\varphi(b)$ holds in the monster model.

We will assume a basic knowledge of stability theory but will give references to statements in [5] or [9] where appropriate. References to statements like A.5, B.10, D.4, etc., always refer to results in [5]. More specifically, we make critical use of the notion of average type of a set of indiscernibles (see [9, p. 88]) and the notion of a strongly regular type. There are several equivalent definitions of strong regularity. We will use the following (see D.13 and D.16).

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DEFINITION. Let $p$ be stationary with $\varphi \in p$. We say $p$ is *strongly regular via* $\varphi$ if there are models $M, N$ with $N \supset M \supset \operatorname{dom} p$ and an $a \in N \backslash M$ such that $t(a/M) = p \mid M$ and for all $b \in N \backslash M$ $\varphi(b)$ implies $t(b/M) = p \mid M$.

Hereafter, we will write "$p$ is SR via $\varphi$" for "$p$ is strongly regular via $\varphi$".

In this paper it is important to bear in mind that strong regularity is a property of the pair $(p, \varphi)$. The usual situation is when $p$ and $\varphi$ have the same Morley rank but this is not required in the above definition. For example, consider a model $M$ of the theory with one equivalence relation with infinitely many infinite classes. Let $q = \{E(x, a)\}$ for some $a \in M$. Then $q$ is SR via $x = x$ even though $q$ forks over $\varnothing$. This example also shows that one formula may "generate" more than one SR type since the unique type over $\varnothing$ is SR via $x = x$.

The next lemma indicates what is implied when $p$ forks over dom $\varphi$. It is never used explicitly but is the motivation behind the main technical lemma, Lemma 5.

LEMMA 1. *Suppose $p$ is SR via $\varphi$ and* dom $\varphi = c$. *If $p \not\perp c$ then $p$ does not fork over $c$.*

PROOF. Suppose towards a contradiction that $p$ forks over $c$. As types in $\omega$-stable theories are based on finite sets, we may assume that $b = \operatorname{dom} p$ is finite. It is easy to show that we may choose $M, N$, and $a$ witnessing the strong regularity of $p$ such that $M$ is a $\aleph_0$-saturated and $N$ is prime over $M \cup a$. Then (1) for every $e \in N \backslash M$, $\varphi(e)$ implies $e \models p \mid M$.

Let $b' \in M$ be such that $b \overset{s}{=} b'(c)$ and $b' \downarrow_c b$; $p' = p_{b'}$. By C.6(i) $p \not\perp p'$. Hence $p' \mid M \not\perp p \mid M$. As $p' \mid M$ is also SR by D.18 it is realized by some $e \in N$. By (1) $e$ must also realize $p \mid M$. However, $b \downarrow_{b'} e$ since $M \downarrow_{b'} e$. Since $b \downarrow_c b'$ we conclude, by the transitivity of nonforking, that $b \downarrow_c e$, contradicting that $p$ forks over $c$. This proves the lemma.

We will also refer to the dimension of a type. If $p$ is strongly regular and $A \supset \operatorname{dom} p$ then $\dim(p, A)$ is the cardinality of a maximal independent subset of $p(A)$. (See page D-5.) Usually we only refer to dimension for types of weight 1. However, for any type $p$ and $A \supset \operatorname{dom} p$, $\dim(p, A)$ is well-defined when $p(A)$ contains an infinite independent subset and $T$ is superstable. We may refer to the dimension of an arbitrary type under these conditions.

In our proof of Theorem A, we never explicitly mention defining schema. Proofs of results found in [1] show the resplendent model $M$ will be homogeneous iff for all $p$ with finite domain $\subset M$, whenever $\dim(p, M)$ is infinite $\dim(p, M) = |M|$. In §1 we prove this when $p$ is SR. In §2 we prove that this is sufficient to prove Theorem A. The recursive definability problem is discussed in §3.

## §1. Dimensions of strongly regular types

This section is devoted to the proof of Theorem 6. In the first two lemmas, we show that if $p \in S(c)$ is SR via $\varphi$ then there is a theory $T'$ recursive in $t(c)$ and containing a predicative $\boldsymbol{I}$ such that $T'$ implies $\boldsymbol{I}$ is an infinite set of indiscernibles with average type SR via $\varphi$.

LEMMA 2. *Let* $\boldsymbol{M}$ *and* $\boldsymbol{I}$ *be new unary predicate symbols,* $\boldsymbol{a}$ *a new constant and* $L' = L \cup \{\boldsymbol{M}, \boldsymbol{I}, \boldsymbol{a}\}$. *There is an* $L'$*-theory* $T'$, *recursive in* $T$, *such that if* $M$, $I$, *and* $a$ *interpret* $\boldsymbol{M}$, $\boldsymbol{I}$, *and* $\boldsymbol{a}$ *in some model of* $T'$ *then* $M \models T$, $I \subset M$ *is an infinite set of indiscernibles and* $t(a/M) = \mathrm{Av}(I/M)$.

PROOF. We let $T'$ consist of the union of the sentences expressing the following:

(i) "$\boldsymbol{M}$ is a model of $T$",

(ii) "$\boldsymbol{I} \subset \boldsymbol{M}$ is an infinite set of indiscernibles",

(iii) $(\forall \bar{y} \in \boldsymbol{M})[\varphi(\boldsymbol{a}, \bar{y}) \rightarrow \bigwedge_{k<\omega}(\exists x_0 \cdots x_{k-1} \in \boldsymbol{I})$
$(\bigwedge_{i<j<k} x_i \neq x_j \wedge \bigwedge_{i<k} \varphi(x_i, \bar{y}))]$;
$\varphi(x, \bar{y}) \in L$

We leave it to the reader to check that $T'$ has the properties claimed in the theorem.

LEMMA 3. *Suppose* $p \in S(c)$ *is SR via* $\varphi$. *Let* $L' \supseteq L$ *contain new unary predicates* $\boldsymbol{M}$ *and* $\boldsymbol{N}$, $\boldsymbol{a}$ *a new constant symbol and constants for c. Then there is an* $L'$*-theory* $T'$, *recursive in* $t(c)$, *such that if* $M$ *and* $a$ *interpret* $\boldsymbol{M}$ *and* $\boldsymbol{a}$ *in some model of* $T'$ *then* $t(a/M)$ *is SR via* $\varphi$.

PROOF. Let $T'$ consist of the sentences expressing

(i) "$\boldsymbol{N} \supset \boldsymbol{M}$ are models of $T(c)$, $c \in M$",

(ii) $\boldsymbol{a} \in \boldsymbol{N} \setminus \boldsymbol{M}$ and $\varphi(\boldsymbol{a})$,

(iii) $(\forall b \in \boldsymbol{N} \setminus \boldsymbol{M})[\varphi(b) \rightarrow \bigwedge_{\psi}(\forall \bar{y} \in \boldsymbol{M})(\psi(\boldsymbol{a}, \bar{y}) \leftrightarrow \psi(b, \bar{y}))]$.

The consistency of $T'$ follows from the existence of a $p$ such that $p$ is SR via $\varphi$. It's not difficult to show $T'$ has the desired properties.

As we remarked in the introduction, we cannot conclude directly that $t(a/M) = p \mid M$. Lemma 1 implies that if we want to ensure $a \downarrow_c M$, we need only require $t(a/M) \not\angle c$. Our experience has shown that for $I \subset M$ indiscernible we must have $\mathrm{Av}(I/M) \not\angle c$ iff $I$ is pairwise $c$-independent. This pairwise independence can be expressed with formulas. In the proof below we won't show that pairwise independence implies $\mathrm{Av}(I/M) \not\downarrow c$; an easier argument contradicts $a \not\downarrow_c M$; but Lemma 1 is the motivation behind the next two lemmas.

LEMMA 4. *Let $M$ be a model with $c \in M$ such that $p \in S(c)$ is SR via $\varphi$. Furthermore, suppose $M$ contains $a_0$, $a_1$ realizing $p$ such that $a_0 \downarrow_c a_1$. Then there is a theory $T'$, recursive in $t(a_0 a_1 c)$, containing a unary predicate $\mathbf{I}$ such that in any model of $T'$ the interpretation $I$ of $\mathbf{I}$ satisfies*: (1) *$I$ is an infinite pairwise $c$-independent set of indiscernibles*, (2) *for some* (*hence, as easily seen, for all*) $N \supset I$, $\mathrm{Av}(I/N)$ *is SR via $\varphi$.*

PROOF. This lemma follows by combining the previous two since we may express recursively in $t(a_0 a_1 c)$ that the 2-type realized by elements of $I$ is $t(a_0 a_1/c)$.

LEMMA 5. *With notation as in Lemma 4, $I$ is independent over $c$.*

PROOF. The theory $T'$ will contain symbols for models $N \supset M$, indiscernibles $I \subset M$ and $a \in N \backslash M$ such that $t(a/M) = \mathrm{Av}(I/M)$ is SR via $\varphi$. If $T'$ doesn't imply $I$ is $c$-independent we can add to $T'$ a sentence implying that $I$ isn't independent and obtain a model where $M$ and $N$ are $\aleph_0$-saturated. To simplify notation assume $c = \varnothing$. Thus, we assume towards a contradiction we have $M, N, I$ and $a$ such that

(i) $N \supset M$ are $\aleph_0$-saturated models, $a \in N \backslash M$, $a \models p$,

(ii) $I \subset M$ is an infinite pairwise independent set of indiscernibles which isn't independent,

(iii) $t(a/M) = \mathrm{Av}(I/M)$ and for all $b \in N \backslash M$, $\varphi(b)$ implies $t(b/M) = t(a/M)$.

Let $\bar{b} \in I$ be such that $t(a/M)$ is based on $\bar{b}$ (see second remark on p. 89 in [9]). Notice that $I \backslash \bar{b}$ is a Morley sequence $t(a/\bar{b})$ and that if $I_0$ is a Morley sequence in $t(a/\bar{b})$ then $I_0 \cup \bar{b}$ is indiscernible. Let $\bar{d} \in N$ be such that

$$\bar{d} \stackrel{s}{\equiv} \bar{b}(a) \quad \text{and} \quad \bar{d} \downarrow_a \bar{b}. \tag{1}$$

Since $I$ is pairwise independent each $b_0 \in \bar{b}$ satisfies $b_0 \downarrow a$, hence each $d_0 \in \bar{d}$ also satisfies $d_0 \downarrow a$. By (1) and by transitivity of nonforking

$$\text{for all } d_0 \in \bar{d}, \qquad d_0 \downarrow \bar{b}. \tag{2}$$

We want to show that $\bar{d} \notin M$. To this end we claim $a \not\downarrow_{\bar{b}} \bar{d}$. Suppose towards a contradiction that $a \downarrow_{\bar{b}} \bar{d}$. Since $t(a/\bar{b}) \simeq t(a/\bar{d})$ they have the same $U$-rank. Thus, $U(a/\bar{b}\bar{d}) = U(a/\bar{b}) = U(a/\bar{d})$ and we have $a \downarrow_{\bar{d}} \bar{b}$. Let $J$ be an infinite

Morley sequence in $t(a/\bar{b}\bar{d})$. Then $J$ is also a Morley sequence in $t(a/\bar{b})$ and $t(a/\bar{d})$. As we remarked above $J \cup \bar{d}$ will be indiscernible. $J$ isn't independent over $\varnothing$ so $J \not\downarrow d_0$ for each $d_0 \in \bar{d}$. However, since we assume $a \downarrow_{\bar{b}} \bar{d}$ we have $J \downarrow_{\bar{b}} \bar{d}$, in particular, $J \downarrow_{\bar{b}} d_0$. Combining this with (2) we obtain $J \downarrow d_0$, a contradiction which proves the claim.

The claim implies that $\bar{d} \not\subset M$. Let $d_0 \in \bar{d}$ be such that $d_0 \in N \backslash M$. Then $d_0$ satisfies $\varphi$ and by (2), $d_0 \downarrow \bar{b}$. This contradicts (ii) and (iii), proving the lemma.

THEOREM 6. *Let $M$ be a resplendent model with $c \in M$ and $p \in S(c)$ SR via $\varphi$ such that $M$ contains $a_0, a_1$ realizing $p$ with $a_0 \downarrow_c a_1$. Then* $\dim(p, M) = |M|$.

PROOF. To the theory $T'$ of Lemma 4, add a statement involving a new function symbol which implies $\boldsymbol{I}$ is equicardinal with the model. Lemma 5 implies $I$ is independent over $c$ witnessing that $\dim(p, M) = |M|$, where $I$ interprets $\boldsymbol{I}$ in an expansion of $M$ to a model of $T'$.

## §2. Dimension and homogeneity

This section is devoted to a proof of Theorem A via Theorem 6. By way of motivation suppose the resplendent $M$ is also $\aleph_0$-saturated. Then Theorem 6 and the theory of regular and SR types developed in Section D of [5] will imply that $M$ is saturated. (D.10 and D.17 are the crucial results.) In this section we relativize the results of $D$ to $\aleph_0$-homogeneous models in $\omega$-stable theories. Throughout this discussion we let $K$ be the class of $\aleph_0$-homogeneous models realizing the same types as some given $\aleph_0$-homogeneous model. As before $T$ is $\omega$-stable.

LEMMA 7. *$K$ has prime models over sets. That is, if $A$ is a subset of some model in $K$ then there is an $M \in K$ such that $A \subset M$ and whenever $N \in K$ and $N \supset A$, $M$ can be embedded into $N$ while fixing $A$.*

PROOF. See [8] or in the recursively saturated case [3].

We'll call $M$ above the *$K$-prime model over $A$* and use *$K$-isolated* for the corresponding notion of isolation. Specifically, we say a type $p \in S(A)$ is $K$-isolated if there is a finite $B \in A$ such that for all $M \in K$ containing $A$, $p \upharpoonright B$ is realized in $M$ and for $a \in M$, $a \models p \upharpoonright B \Rightarrow a \models p$.

LEMMA 8. *Suppose $M \subset N$ are in $K$. Then for all $b \in N \backslash M$ there is an infinite set of indiscernibles $I \subset M$ such that* $t(b/M) = \mathrm{Av}(I/M)$.

PROOF. See [6] or [4].

DEFINITION. We say a model $M \in K$ is *K-model homogeneous* if whenever $N_i \subset M$, $|N_i| < |M|$ and $N_i \in K$ for $i = 0, 1$; $f : N_0 \simeq N_1$; and $p \in S(N_0)$ is realized in $M$; $f(p)$ is also realized in $M$.

LEMMA 9. *For $M \in K$ uncountable, $M$ is homogeneous iff $M$ is $K$-model homogeneous.*

PROOF. The proof is essentially the same as the proof of lemma 9 in [1], substituting $K$-prime for prime.

LEMMA 10. *Given $M \in K$ and $N \supset M$ $K$-prime over $M \cup a$ for some $a$ there is a $\bar{c} \in N \backslash M$ such that* (i) *each $c_i$ is SR over $M$,* (ii) *$\bar{c}$ is $M$-independent,* (iii) *$N$ is $K$-prime over $M \cup \bar{c}$. In particular, $t(a/M \cup \bar{c})$ is $K$-isolated.*

PROOF. The proof of this lemma is almost identical to the proof of D.10. Notice that this is stronger than D.17 in requiring that $\bar{c}$ be $M$-independent. This is possible since whenever $b, B \subset N$ are finite and $t(b/B) \vdash t(b/M \cup B)$ we conclude that $t(b/M \cup B)$ is $K$-isolated.

THEOREM 11. *Suppose $M$ is $\aleph_0$-homogeneous and whenever $p$ is an SR type over a subset of $M$ with* $\dim(p, M)$ *infinite we have* $\dim(p, M) = |M|$. *Then $M$ is homogeneous.*

PROOF. Let $K$ be the class of $\aleph_0$-homogeneous models realizing the same types as $M$. Wlog, $M$ is uncountable. By Lemma 9 it suffices to show $M$ is $K$-model homogeneous. Let $N_0, N_1, f$ and $p$ be as in the definition of $K$-model homogeneity. Let $a \in M$ realize $p$. By Lemma 10 there is $\langle c_i \rangle_{i<n} \in M$ an $N_0$-independent sequence of elements SR over $N_0$ such that $t(a/N_0 \cup \bar{c})$ is $K$-isolated. For $i < n$ let $q_i$ be an SR type over a finite subset of $N_0$ such that $t(c_i/N_0) = q_i \mid N_0$. By Lemma 8, $q_i$ has infinite dimension in $N_0$. Thus $f(q_i)$ has infinite dimension in $N_1$. By hypothesis $\dim(f(q_i), M) = |M|$. Choose $\langle d_i \rangle_{i<n}$ as follows. Let $d_0 \in M$ realize $f(q_0) \mid N_1$. If $d_0, \ldots, d_k$ have been chosen, let $d_{k+1} \in M$ realize $f(q_{k+1}) \mid N_1 d_0 \cdots d_k$. This is possible since $|N_1 d_0 \cdots d_k| < \dim(f(q_{k+1}), M)$.Thus, we have $\bar{d} \in M$ such that $f(t(\bar{c}/N_0)) = t(\bar{d}/N_1)$. Since $t(a/N_0 \cup \bar{c})$ is $K$-isolated, we can find $b \in M$ such that $f(t(a\bar{c}/N_0)) = t(b\bar{d}/N_1)$. Thus $b \models f(p)$, proving the theorem.

Combining Theorems 6 and 11 we obtain

THEOREM A. *Every resplendent model of an ω-stable theory is homogeneous.*

For applications of this theorem see section 4 of [1].

## §3. Recursive definability

In the past there have been several different formulations of what it means for a stable theory to satisfy "recursive definability". The relative strengths of these notions are still unclear. Here we list a few possible definitions for $\omega$-stable $T$.

*RD*0 (Buechler). For all finite $c$ and $p \in S(c)$ stationary there is a map $d$ from $L$-formulas to formulas over $c$ such that (i) for all $X$ containing $c$, $p \mid X$ is defined by $d$, (ii) $d$ is recursive in $t(ac)$ for some $a \models p$.

*RD*1 (Baldwin). For all $a$ and $c$ such that $t(a/c)$ is stationary there is a recursive sequence $\{\Psi_k : k < \omega\}$ of $\Sigma_1^1$ formulas such that for all $k, a'$ and $b_1, \ldots, b_k$, $\Psi_k(a', \bar{b}, a, \bar{c})$ holds iff (i) $t(a/c) = t(a'/c)$ and (ii) $a' \downarrow_c \bar{b}$.

We say a type $p \in S(M)$ is *finitely realized* in $M$ if for all $c \in M$, $p \restriction c$ is realized in $M$.

*RD*2 (Knight [2]). Suppose $M$ is recursively saturated and $p \in S(M)$ is finitely realized in $M$. Then there is a finite $c \in M$ such that for all $a \supset c$, $p \mid a$ is recursive in $t(a)$.

*RD*3 (Knight–Lachlan [3]). Suppose $M$ is recursively saturated and $p \in S(M)$ is finitely realized in $M$. Then there is a finite $c \in M$ and $d$ a definition of $p$ over $c$ such that $d$ is recursive in $t(c)$.

The strongest of these notions is RD0. It will be true whenever the rank function $p, \varphi \rightarrow R(p, \varphi, 2)$ is recursive in $t(ac)$. To indicate the strength of RD0 we will show it implies a variant of RD1; namely

*RD1′* Suppose $p = t(a/c)$ is stationary. Then for all $k < \omega$ there is a $\Sigma_1^1$ formula $\Psi_k$ over $ac$ such that

(i) $\{\Psi_k : k < \omega\}$ is recursive in $T(ac)$,
(ii) $\Psi_k(a', b)$ holds in the monster iff $t(a'/bc) = p \mid b$.

Suppose RD0 holds. For an arbitrary $k < \omega$ let $\{\varphi_j : j < \omega\}$ be a recursive list of formulas of the form $\varphi(x, y)$, where $y = y_1 \cdots y_k$. By RD0 the set $\{d\varphi_j : j < \omega\}$ is recursive in $T(ac)$. Let $\Gamma(x, y)$ be the type recursive over $ac$ expressing

(1) $\psi(x, c) \leftrightarrow \psi(a, c)$; $\psi \in L$,
(2) $\varphi_j(x, y) \leftrightarrow d\varphi_j(y)$; $j < \omega$.

We can effectively find a $\Sigma_1^1$ formula $\Psi_k$ which is equivalent to $\Gamma$ on resplendent models (see proof #1 of III.1.9 in [7]). The set $\{\Psi_k : k < \omega\}$ witnesses RD1′. Proving RD0 $\Rightarrow$ RD1 requires a little more about definitions.

Lemma 1.1 of [3] implies that RD2 and RD3 are equivalent. Combining this fact with Theorem A and lemma 3.3 of [2] proves

THEOREM B. *If $T$ is $\omega$-stable, then* RD3 *holds.*

Notice that Theorem B does not immediately imply RD0 or RD1. The best result appears to be the following theorem. That we can't necessarily choose $e = c$ indicates why RD3 doesn't imply RD0.

THEOREM 12. *Let $T$ be $\omega$-stable and $p \in S(c)$ stationary. Then there is a defining scheme $d$ over $c$ such that* (i) *for all $A$ containing $c$ $d$ defines $p \mid A$ and* (ii) *for some $e \supset c$ $d$ is recursive in $t(e)$. Furthermore, we may choose $e$ from any recursively saturated model $M$ containing $c$ and $I$ an infinite Morley sequence in $p$.*

PROOF. Let $M$ be as in the statement of the theorem. Let $q = \mathrm{Av}(I/M)$. Then $q$ is finitely realized in $M$ so there is an $e \supset c$ such that $q$ is defined over $e$ by some $d_0$ recursive in $t(e)$. Since $q$ is based on $c$ it's definable over $c$. Thus, for all $\varphi$, $d_0\varphi$ is equivalent to a formula over $c$. We define $d$, a map from formulas to formulas over $c$ by: $d$ is the first formula $\gamma$ (in some list of formulas over $c$) such that $\forall \bar{y}(\gamma(\bar{y}, c) \leftrightarrow d_0\varphi(\bar{y}, e))$. This statement appears to be recursive in $t(e)$.

As was noted in theorem 4 of [1], $d$ will satisfy (i).

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